Exercise 4

Hand-in your results electronically latest Sep. 28, 2016, 24:00h.

This lab has 5 task.

Task 1
Perform the MATLAB/Python experiments in the course book Exercise 12.3

Task 2
Solve Exercise 13.3 in the course book.

Task 3
The condition number of a matrix gives a sharp estimate of the sensitivity of $x$ with respect to perturbations of $b$ when solving $Ax = b$, this means there exists a right hand side $b$ and a perturbation $\delta b$ such that

$$\frac{\|\delta x\|_2}{\|x\|_2} = \kappa_2(A) \frac{\|\delta b\|_2}{\|b\|_2}$$

(Note the equal sign!). Give a vector pair $(b, \delta b)$ for which this equality holds. Hint, express these vectors in terms of left singular vectors.

Task 4

Hilbert matrices are notoriously ill conditioned. Verify your result from Task 3 by solving a linear system with a $50 \times 50$ Hilbert matrix and a worst case $b$ and $\delta b$. Hilbert matrices and their exact inverses can be constructed in MATLAB by hilb and invhilb and in Python by the commands scipy.linalg.hilbert and scipy.linalg.invhilbert.

Task 5

Let $A$ be a $m \times n$ matrix. Prove the statement

$$\mathcal{N}(A^T)^\perp = \text{Range}(A)$$

\[1\]
You need in Python to have Scipy version 0.10 or higher installed.