Numerical Methods for Differential Equations

Assignment 1

Problem 1
The trapezoidal rule is given by
\[ y_{n+1} = y_n + \frac{\Delta t_n}{2} (f_n + f_{n+1}). \]
Prove: The trapezoidal rule is A-stable.

Problem 2
Consider the method
\[ y_{n+1} = y_n + \Delta t (a_1 f(t_n, y_n) + a_2 f(t_n + b_1 \Delta t, y_n + b_2 \Delta t f(t_n, y_n))). \]
This is a so called explicit 2-stage Runge-Kutta method.

- Determine conditions on the parameters \(a_j, b_j \in \mathbb{R}\), such that the method is consistent of first order.
- Determine the parameters, such that the method is consistent of order 2.

Problem 3
Consider the equation
\[ \dot{y}(t) = -10y(t), \quad y(0) = 1. \]
Determine the maximal time step for which we should have stability for the explicit Euler method. Solve the problem (using a code of yours) with the explicit Euler method and the implicit Euler method until \(t = 2\), with three different step sizes each. One should be directly on the stability boundary of the explicit Euler method, one beyond and one inside. Describe your results.

Problem 4
Use the Python routines scipy.integrate.RK45 and scipy.integrate.BDF, respectively the MATLAB solvers ode45 and ode15s, to solve the initial value problem (IVP)
\[ \begin{align*}
\dot{c}_A(t) &= -0.04 c_A(t) + 10^4 c_B(t) c_C(t), \\
\dot{c}_B(t) &= 0.04 c_A(t) - 10^4 c_B(t) c_C(t) - 3 \cdot 10^7 c_B^2(t), \\
\dot{c}_C(t) &= 3 \cdot 10^7 c_B^2(t), \\
c_A(0) &= 1, \quad c_B(0) = c_C(0) = 0,
\end{align*} \]
for the time interval \(t \in [0, 100]\). Plot the solution for each of the solvers. What happens? How many time steps did the solvers need?

Return: Tuesday, November 13th, 8:00