This describes the second bigger programming project in the course, devoted to some classical optimization algorithms.
Be prepared to have this project completed latest on Wednesday, October 03.
This assignment has 12 tasks.

**Task 1**

Design an optimization problem class in Python. Its constructor should take an objective function as input, optionally also its gradient can be provided.

**Task 2**

Design a general optimization method class. It should be generic for special kinds of Quasi-Newton methods. Derive from this method class classes for special methods in the next tasks.

**Task 3**

Implement classical Newton’s method for finding a minimum of the objective function. The linear solver within Newton’s method should be based on Cholesky’s method. Approximate the Hessian by finite differences and a symmetrizing step: $G := \frac{1}{2}(\tilde{G} + \tilde{G}^T)$. Raise an exception if the Hessian is not positive definite.

**Task 4**

Provide Newton’s method with exact line search method.

**Task 5**

Test the performance of this method on the Rosenbrock function

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2.$$
Rosenbrock Function: \( f(x, y) = (1 - x)^2 + 100(y - x^2)^2 \)

Figure 1: A contour plot of Rosenbrock’s function and steps towards its minimum.

Demonstrate its behavior also in a way, comparable to the right picture in Fig. 1.

Task 6

Write an inexact line search method based on the Goldstein/Wolfe conditions (Algorithm: Fletcher pp.34ff in the additional course material)

Task 7

Test this seperately from an optimization method on Rosenbrock’s function and use the parameters given on p.37 in the book mentioned above.

Task 8

Provide Newton’s method with the inexact line search as an alternative to the exact one.

Task 9

Derive from Newton’s method four classes of Quasi Newton methods:

- Simple Broyden rank-1 update of \( G \) and applying Sherman-Morisson’s formula ("good Broyden")
- Simple Broyden rank-1 update of \( H = G^{-1} \) ("bad Broyden")
- DFP rank-2 update
- BFGS rank-2 update
Task 10

Download from the course’s webpage the testexample chebyquad_problem.py. It contains the objective function chebyquad and its gradient.

The objective function in mathematical terms reads:

\[
f(x) = \sum_{i=0}^{n} \left( \frac{1}{n} \sum_{j=0}^{n} T_i(2x_j - 1) - \int_{0}^{1} T_i(2\xi - 1)d\xi \right)^2 \quad x = [x_0, \ldots, x_n] \quad x_j \in [0, 1]
\]

where \(T_i(t)\) is the \(i\)-th degree Chebyshev polynomial and \(t \in [-1, 1]\).

Task 11

Minimizing chebyquad corresponds to finding a set of optimal points \(x_j\) such that the mean value in (1) approximates best the corresponding integrals. Use your code to compute these points for \(n = 4, 8, 11\). Compare your results with those obtained from scipy.optimize.fmin_bfgs.

Task 12

The matrix \(H^{(k)}\) of the BFGS method should approximate \(G(x^{(k)})^{-1}\), where \(G\) is the Hessian of the problem. Study the quality of the approximation with growing \(k\).

Lycka till!