The purpose of this training assignment is to experiment more with lists, for-loops and to work with simple functions.

This assignment has 2 tasks.

Task 1

The point $x$ with the property $x = \sin(x) - ax + 30$ is called a fixed point of the function $f(x) = \sin(x) - ax + 30$. It can be computed with the so-called fixed point iteration:

$$x^{(i+1)} = \sin(x^{(i)}) - ax^{(i)} + 30,$$

where the superscript $(i)$ denotes an iteration counter. Here you find a piece of Python code which performs the first 200 steps of this iteration:

```python
x=0.5
a=0.5
for i in range(200):
    x=sin(x)-a*x+30
    print('The result after {num} iterations is {res}'.format(num=i,res=x))
```

Modify the code in such a way that it stops iterating as soon as $|x^{(i+1)} - x^{(i)}| < 10^{-8}$. Furthermore it should print a little message if this condition was not met within 200 iteration steps. (The absolute value is computed in Python by the function `abs` and $10^{-8}$ is written as `1.e-8`.)

Test your code with $a = 0.5$ and $a = 8$.

Task 2

Plot the function with $a = 0.5$ from the previous task in the range $x \in [5, 30]$ and in the same figure plot also the function $y = x$. Do you expect that the function has several fixed points?

Hint: You may want to use the `linspace` command to generate the $x$-values. (Note, `linspace` returns a datatype `array`, which we treat here as if it would be a list. The differences between these datatypes will become clear later.)
**Task 3**

Let

\[ x_n = \frac{(\sin n)^2}{n} \quad \forall n > 0. \]

It is easy to show that \( \lim_{n \to \infty} x_n = 0 \). Create a list containing all the elements of this sequence \( x_n \) for all \( n \) until \( x_n < 10^{-9} \). How long is this list?

**Task 4**

Consider the sequence:

\[ x_{n+1} = 0.2 x_n - \alpha (x_n^2 - 5) \quad \text{with} \quad x_0 = 1 \]

for \( \alpha \) successively equal to -0.5, +0.5, -0.25, 0.25.

- Check the convergence; if the sequence converges, print the message
  
  Sequence converged to \( x = \text{<the value you got>} \)

  otherwise print

  No convergence detected.

- Check whether there are negative elements in the sequence

  Hint: If \( |x_n - x_{n-1}| < 10^{-9} \) consider a sequence to be convergent.

**Task 5**

For \( \alpha = 0.5 \) in the last task you got positive as well as negative elements. Construct two lists, one which contains the positive elements of the sequence and one which contains the negative elements.

**Task 6**

Write a function which has \( \alpha \) as input. The function should perform the convergence test of the sequence given above. It should return \texttt{True} if the sequence converged within 30 iterations and return \texttt{False} if it didn’t.

Test your function with the same values of \( \alpha \) as in Task 4. Try it also with \( \alpha = 1 \).

What happens?

**Task 7**

Give your function a second input argument \( x_0 \) so that it allows you to test convergence for different \( \alpha \) and different starting values \( x_0 \).
Task 8

Give your functions two additional output parameters: \texttt{pos} and \texttt{neg}. One of them should contain the positive elements of the sequence and the other the negative ones. If there are only positive elements in the sequence \texttt{neg} should be an empty list and vice versa.

Good luck!