The purpose of this training assignment is to experiment more with lists, for-loops and to work with simple functions.
This assignment has 8 tasks.

Task 1

The point $x$ with the property $x = \sin(x) - ax + 30$ is called a fixed point of the function $f(x) = \sin(x) - ax + 30$. It can be computed with the so-called fixed point iteration:

$$x^{(i+1)} = \sin(x^{(i)}) - ax^{(i)} + 30,$$

where the superscript $(i)$ denotes an iteration counter. Here you find a piece of Python code which performs the first 200 steps of this iteration:

```python
x = 0.5
a = 0.5
for i in range(200):
    x = sin(x) - a*x + 30
    print('The result after {num} iterations is {res}'.format(num=i, res=x))
```

Modify the code in such a way that it stops iterating as soon as $|x^{(i+1)} - x^{(i)}| < 10^{-8}$. Furthermore it should print a little message if this condition was not met within 200 iteration steps. (The absolute value is computed in Python by the function `abs` and $10^{-8}$ is written as `1.e-8`.)

Test your code with $a = 0.5$ and $a = 8$.

Task 2

Plot the function with $a = 0.5$ from the previous task in the range $x \in [5, 30]$ and in the same figure plot also the function $y = x$. Do you expect that the function has several fixed points?

Hint: You may want to use the `linspace` command to generate the $x$-values. (Note, `linspace` returns a datatype `array`, which we treat here as if it would be a list. The differences between these datatypes will become clear later.)
Task 3

Let
\[ x_n = \frac{(\sin n)^2}{n} \quad \forall n > 0. \]

It is easy to show that \( \lim_{n \to \infty} x_n = 0 \). Create a list containing all the elements of this sequence \( x_n \) for all \( n \) until \( x_n < 10^{-9} \). How long is this list?

Task 4

Consider the sequence:
\[ x_{n+1} = 0.2 x_n - \alpha (x_n^2 - 5) \] with \( x_0 = 1 \)

for \( \alpha \) successively equal to -0.5, +0.5, -0.25, 0.25.

- Check the convergence; if the sequence converges, print the message
  
  Sequence converged to \( x = \text{<the value you got>} \)

  otherwise print
  
  No convergence detected.

- Check whether there are negative elements in the sequence

  Hint: If \( |x_n - x_{n-1}| < 10^{-9} \) consider a sequence to be convergent.

Task 5

For \( \alpha = 0.5 \) in the last task you got positive as well as negative elements. Construct two lists, one which contains the positive elements of the sequence and one which contains the negative elements.

Task 6

Write a function which has \( \alpha \) as input. The function should perform the convergence test of the sequence given above. It should return True if the sequence converged within 30 iterations and return False if it didn’t.

Test your function with the same values of \( \alpha \) as in Task 4. Try it also with \( \alpha = 1 \). What happens?

Task 7

Give your function a second input argument \( x_0 \) so that it allows you to test convergence for different \( \alpha \) and different starting values \( x_0 \).
Task 8

Give your functions two additional output parameters: pos and neg. One of them should contain the positive elements of the sequence and the other the negative ones. If there are only positive elements in the sequence neg should be an empty list and vice versa.

Good luck!