This assignment has 6 tasks.

Exercises

For these tasks you need knowledge about vectors and arrays, i.e. the basics from Chapter IV.2 and the first sections of Ch. V

Task 1

Write a function, which takes a matrix as parameter. It should check if this matrix is symmetric. The function should return 1 for a symmetric matrix, −1 for a skew-symmetric matrix and 0 otherwise. Test your function.

Task 2

Write a function, which takes two vectors as parameters. It should check if these vectors are orthogonal. If they are orthogonal it should return True, otherwise False.
Don’t forget to provide your function with a docstring.
Test your function.

Task 3

Write a function, which takes a vector as parameter and which returns the corresponding normalized vector, i.e. \( \frac{x}{\|x\|} \). Write two variants of this program: one in which you compute the norm (use the 2-norm) of the vector by yourself and another, which uses the function norm from the module scipy.linalg.
Recall that to be able to use Scipy’s norm-function you must have the line

```python
from scipy.linalg import norm
```

at the start of your program. Note that (of course) you cannot then call your own function norm too. If this is inconvenient, one can use
from scipy.linalg import norm as scnorm

to give Scipy’s norm-function the new name scnorm.

**Task 4**

Show experimentally that the inverse of a rotation matrix is its transpose.
*Hint: B is the inverse of A if AB = BA = I, the identity matrix.*

Note, in 2D a rotation matrix has the form

\[
\begin{pmatrix}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{pmatrix}
\]

where $\alpha$ can be any angle.

**Task 5**

*(If you don’t know eigenvalues (yet) skip this task)* Construct a $20 \times 20$ matrix with the value 4 on its diagonal and the value 1 on its sub- and super-diagonal. The rest of the matrix is zero. Compute its eigenvalues. *(Use the function eig from the module scipy.linalg)*. You might also want to check the function diag for this task

**Task 6**

*(If you don’t know eigenvalues (yet) skip this task)* Change in the above task the matrix in such a way that all the elements of its subdiagonal instead have the value $-1$. How are the eigenvalues affected by this change?