You should upload your solution for this homework at the latest on 2018-02-20

This assignment has 4 tasks.

**Task 1**

Modify the divided difference program you wrote in Homework 1 in such a way, that you can also provide interpolation data for higher derivatives (Hermite interpolation). Use it for working out the example for interpolating \( f(x) = \text{sinc}(x) = \frac{\sin(x)}{x} \) which was given in the lectures.

**Task 2**

Consider Runge’s function \( f(x) = \frac{1}{1+25x^2} \) and interpolate it in \([-1, 1]\) on
- an equidistant grid with an increasing number of grid points
- a grid of your choice with an increasing number of grid points
- a grid based on the zeros of Tschebychev polynomials.

Plot the original curve together with the respective interpolation polynomial. Plot also the error function (difference between two functions).

**Task 3**

We want to approximate \( f(x) = x^2 \) on \([0, 1]\) by a linear function \( s_\xi(x) = \xi \cdot x \), \( \xi \in \mathbb{R} \) with respect to the three norms \( \| \cdot \|_p \) with \( p = 1, 2, \infty \).

Compute first for all three cases the distance function

\[
\eta_p(\xi) := \| s_\xi - f \|
\]

and compute then the respective best approximation \( s_\xi^* \).

(Task 3.68 in Irso).
Let $\mathcal{F}$ be a linear space with a norm $\| \cdot \|$, which is not strictly convex. Show that there exists a function $f \in \mathcal{F}$ and a subspace $\mathcal{S} \subset \mathcal{F}$, so that $f$ has different best approximations $s_1^*$ and $s_2^*$ in $\mathcal{S}$, i.e.,

$$\eta(f, \mathcal{S}) = \|s_1^* - f\| = \|s_2^* - f\| \text{ with } s_1^* \neq s_2^*$$

Hint: Choose suitable, different $f_1, f_2 \in \mathcal{F}$ with $\|f_1\| = \|f_2\| = 1$ and $\|f_1 + f_2\| = 2$. Consider now $f = \frac{1}{2}(f_1 + f_2)$ and $\mathcal{S} = \{\alpha(f_1 - f_2) | \alpha \in \mathbb{R}\} \subset \mathcal{F}$.  

(Task 3.72 in Irsoa).