You should upload your solution for this homework at the latest on 2018-03-06

This assignment has 4 tasks.

Task 1

Proof the Theorem on Slide 3.14 (about strictly convex norms)

Task 2

Let $\| \cdot \|$ be an Euclidean norm (inner product norm). Express $\| \cdot \|'(u, v)$ in terms of the inner product.

Task 3

Compute and plot the polynomial $p_5$ which best approximates the function $f(x) = \arctan(x)$ in the interval $[-1, 1]$. Make three different approaches

1. Use a monomial basis for this task, set up a Hilbert matrix and solve for the coefficients. Use the inner product $(f, g) = \int_{-1}^{1} f(x)g(x)dx$.

2. Use Legendre polynomials as a basis and the same inner product.

3. Use the inner product $(f, g) = \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}} f(x)g(x)dx$ instead and use Chebychev polynomials as a basis.

To compute integrals use SciPy’s command `quad`, i.e. don’t make symbolic computations.

Task 4

Let $\mathcal{P}^n$ be the space of all polynomials of max degree $n$ and assume that $x_i, i = 0 : n$ are given points. Show that

- $(p, q) = \sum_{i=0}^{n} p(x_i)q(x_i)$ is an inner product on $\mathcal{P}^n$
• \((p, q) = \sum_{i=0}^{k} p(x_i)q(x_i)\) with \(k < n\) is no inner product on \(\mathcal{P}^n\).

Good luck!